# AP Statistics – Chapter 11 Notes: Testing a Claim

## 11.1: Significance Test Basics

**Null and Alternate Hypotheses**

The statement that is being tested is called the **null hypothesis ($H_0$)**. The significance test is designed to assess the strength of the evidence against the null hypothesis. Usually the null hypothesis is a statement of "no effect," "no difference," or no change from historical values.

The claim about the population that we are trying to find evidence for is called the **alternative hypothesis ($H_a$)**. Usually the alternate hypothesis is a statement of "an effect," "a difference," or a change from historical values.

**Test Statistics**

To assess how far the estimate is from the parameter, standardize the estimate. In many common situations, the test statistics has the form

$$\text{test statistic} = \frac{estimate - parameter}{\text{standard deviation of the estimate}}$$

**Z-test for a Population Mean**

The formula for the z test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ where $\mu_0$ is the value of specified by the null hypothesis. The test statistic *z* says how far $\bar{x}$ is from $\mu_0$ in standard deviation units.

**P-value**

The p-value of a test is the probability that we would get this sample result or one more extreme if the null hypothesis is true. The smaller the p-value is, the stronger the evidence against the null hypothesis provided by the data.

**Statistical Significance**

If the P-value is as small as or smaller than alpha, we say that the data are statistically significant at level alpha. In general, use alpha = 0.05 unless otherwise noted.

## 11.2: Carrying out Significance Tests

**Follow this plan when doing a significance test:**

1. Hypotheses: State the null and alternate hypotheses
2. Conditions: Check conditions for the appropriate test
3. Calculations: Compute the test statistic and use it to find the p-value
4. Interpretation: Use the p-value to state a conclusion, in context, in a sentence or two

**Conditions for Inference about a Population Mean**

- **SRS** - Our data are a simple random sample (SRS) of size *n* from the population of interest. This condition is very important.
- **Normality** - Observations from the population have a normal distribution with mean $\mu$ and standard deviation $\sigma$ or a sample size of at least 30.
- **Independence** - Population size is at least 10 times greater than sample size

## 11.3: Use and Abuse of Tests

**There are four important concepts you should remember from this section:**

1. *Finding a level of significance* (read p. 717) – if you need to be very sure that the null hypothesis is false, use a lower level of significance such as .01 instead of .05.
2. *Statistical significance does not mean practical importance* (see example 11.13) – it is possible for a test to show that an outcome is rare probabilistically, but that the results do not mean anything from a practical standpoint.
3. *Statistical inference is not valid for all sets of data* (see example 11.16) – unless the data is a random sample from the population of interest, it is likely that data is biased and thus a significance test on that data will also be biased.
4. *Beware of multiple analyses* (see example 11.17) – if a large number of significance tests are conducted on multiple samples, probability alone tells us that some may appear to be significant by random chance. For example, at a significance level of 5%, it is likely that 5% of such tests will be found significant even though they aren't.

## 11.4: Using Inference to Make Decisions

**Type I and Type II Errors**

There are two types of errors that can be made using inferential techniques. In both cases, we get a sample that suggests we arrive at a given conclusion (either for or against $H_0$). Sometimes we get a bad sample that doesn't reveal the truth.

Here are the two types of errors:

**Type I** – Rejecting the Ho when it is actually **true** (a false positive)
**Type II** – Accepting the Ho when it is actually **false** (a false negative)

Be prepared to write, in sentence form, the meaning of a Type I and Type II error in the context of the given situation. The **probability of a Type I error** is the same as alpha, the significance level. <u>You will not be asked to find the probability of a Type II error</u>.

**Power of a Test**

The power of a test is the probability that we will reject Ho when it is indeed false. Another way to consider this is that power is the strength of our case against Ho when it is false. The farther apart the truth is from Ho, the stronger our power is. Also, increasing sample size lowers our chance of making a Type II error, thus increasing power.